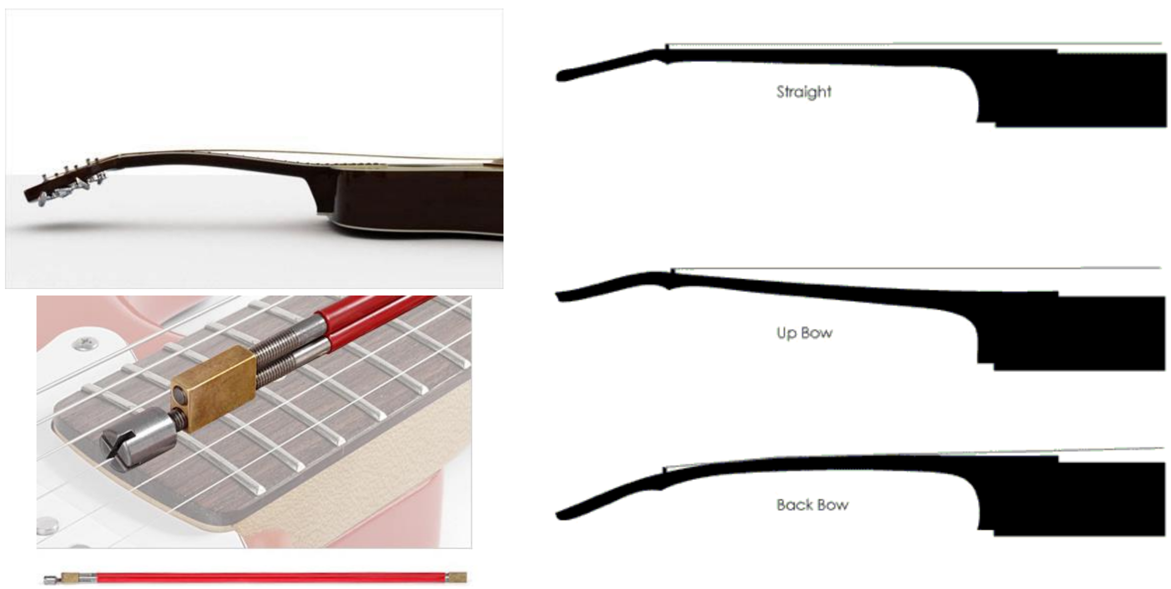
TAM 251 Worksheet 14

Objectives:

- Demonstrate broad application of beam bending analysis (here, a musical instrument)
- From internal axial loads and deformation, calculate bending moments and deflection
- Use superposition to determine beam deflections

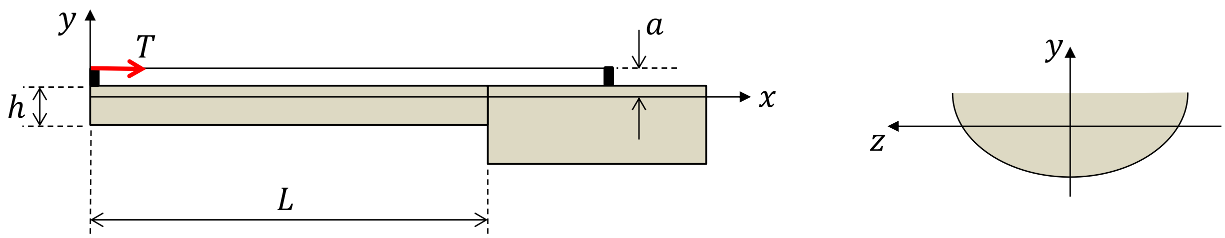
Introduction:

A guitar neck can bow up or down when strings are tightened. To compensate for this bending and to minimize even very small deflections, an adjustable truss rod is inserted in the guitar neck.



In this worksheet, we will first consider the bending of the guitar neck alone due to the tension of the string. Then, we will consider forces within the truss rod alone. Finally, the effects of each will be combined using superposition to attain the solution for the full system.

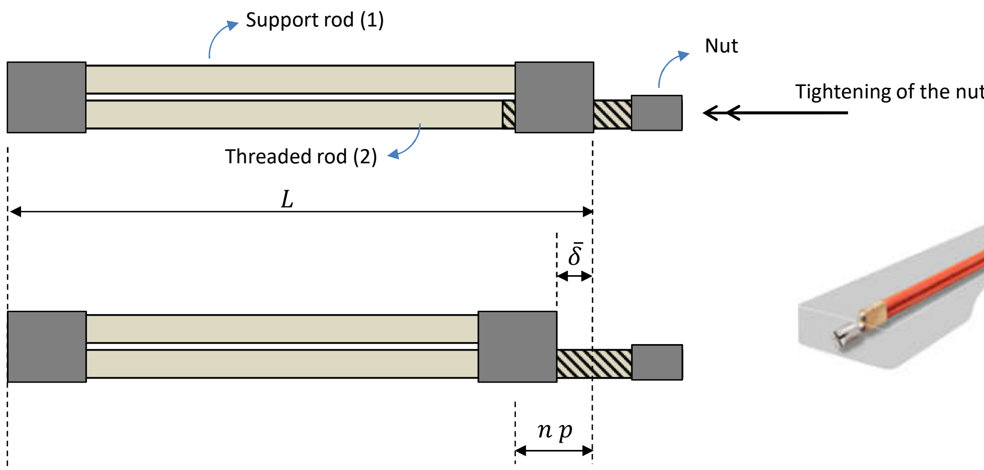
We first consider the bending of the guitar neck in the absence of the truss rod. Assume the neck has a constant cross-sectional area, as illustrated below (no tapering), with flexural rigidity denoted by $(EI)_w$. Model the neck as a cantilevered beam fixed to the guitar body, so that $y_w(L) = y'_w(L) = 0$, where $y_w(x)$ is the deflection of the wooden neck. The string tension is given by T. Note that, as usual, the x-axis is located at the neutral axis of the guitar neck.



a) Determine the internal bending moment of the guitar neck due to the string tension T.

b) Determine the deflection of the guitar end at $x = 0$, $y_w(0)$, by solving $(EI)_w \, y''_w(x) = M(x)$.

We now look at the geometry of deformation of the truss rod alone (before inserting it into the neck cavity). In this work, we consider double action truss rods (albeit one that has been simplified, with threading in fewer places).



We must understand the resulting geometry and deformation when the nut is tightened. Let us assume that the left end of the rod is fixed.

If the support rod was removed, and the remaining threaded rod advanced by n turns, then the working length of the threaded rod would decrease by a distance np, where p is the pitch of the thread.

The support rod (1), when connected, will not deform the full distance np, since it has finite stiffness. Rather, its (compressive) deformation will be represented by $\delta_1 = -\bar{\delta}$ (as shown in the figure). The threaded rod (2) must stretch to accommodate this deformation $\bar{\delta}$.

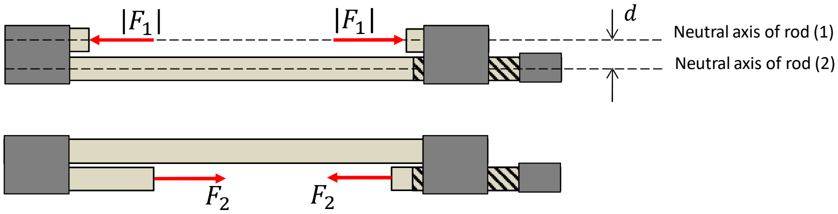
c) Determine the (tensile) deformation of the threaded rod (2), δ_2, in terms of n, p, and $\bar{\delta}$

d) Combine the equation from part c) and those given above to get the compatibility equation—i.e. an equation that relates the deformation of rods (1) and (2).

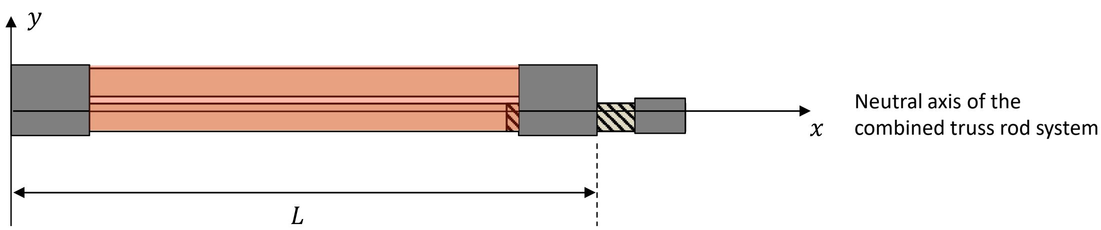
e) Substitute the force-deformation relation into your result from d). The threaded rod (2) has stiffness k_2 and the support rod (1) has stiffness k_1. Use F_1 to represent the internal axial force in rod (1) and F_2 to represent the internal axial force in rod (2).

f) Use equilibrium to obtain the internal forces F_1 and F_2.

As you can see, tightening the nut will apply a compressive force to the threaded rod and a tensile force to the support rod.



g) If we now consider the two rods as one beam system (see figure below), what is the most appropriate representation of the loading condition at each end of the beam due to the tightening of the nut (i.e. how are the pairtos of rod forces best represented as simple loads)? Draw this loading condition in the figure below and obtain an expression for it.



In summary, tightening the nut provides a negative moment that counteracts the neck deflection due to the tension and removes up-bow; on the other hand, turning the nut in the other direction provides a positive moment, which allows the neck to relax into an upward-bowing shape again (especially when helped by the string's pull).

h) When placed inside the guitar neck cavity, we model the truss rod as fixed at the body of the guitar ($x = L$). Therefore, when analyzing the deflection of the truss rod system $y_r(x)$, we apply the following boundary conditions: $y_r(L) = y_r'(L) = 0$. Note that the threaded rod and the support rod have different curvatures, however, since these curvatures are very small, we can approximate them by the curvature of the equivalent truss rod system with flexural rigidity $(E\,I)_r$. Determine the deflection of the truss rod system at $x = 0$ when the bending moment obtained in part g) is applied.

i) The truss rod is now placed inside the guitar neck to compensate bending due to string tension. In this work, we assume the truss rod fits snugly in the cavity and that the resulting displacement can be obtained by using the principle of superposition. Obtain an expression for the number of turns n that will make the displacement of the end of the guitar neck equal to zero (represented by the equation $y(0) = y_r(0) + y_w(0) = 0$). Your result should be a function of the given quantities: T, p, d, k_1, k_2, $(E\,I)_r$ and $(E\,I)_w$.