TAM 251 Worksheet 8

Objectives:

- Sketch a free-body diagram
- Determine internal moments and forces
- Draw shear force and bending moment diagrams
- Evaluate allowable force and maximum shear stress

Introduction:

You are designing a custom tall bike with some friends at the Bike Project of Urbana-Champaign. The head mechanic has a custom, flashy, elongated spindle that you are considering using on the bike... but only if it can safely handle pedaling loads.

Your friends have chosen you, the engineer, to determine whether or not the custom spindle can be used safely. For this analysis, you've asked your friend Tam to pull some technical diagrams of the spindle-crank-pedal system, which are provided in the appendix.

Figure 1: Typical commuter bike with spindle-crank-pedal system highlighted.

Figure 2: Tall bike.

Figure 3: Person mounting a tall bike.

Appendix

The resultant load from pedaling is represented as a concentrated force P at the center of the pedal. Assume there is no force applied to the other pedal (not shown).

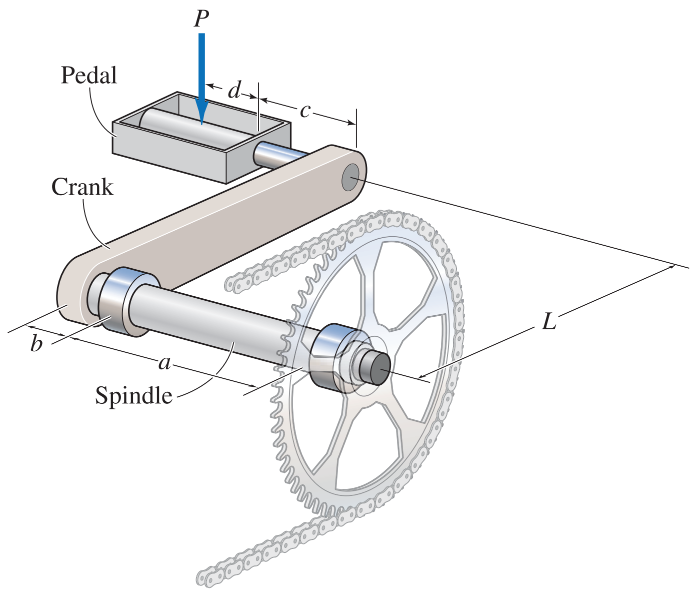


Figure 4: The spindle-crank-pedal system.

$a = 70$ mm
$b = 25$ mm
$c = 9.5$ mm
$d = 40$ mm
$L = 170$ mm

Figure 5: Dimensions.

Assume that the bearings are aligned and set in the frame of the bike, keeping the spindle in equilibrium while exerting only forces (not moments). Therefore, as illustrated in Figure 7, the bike spindle can be represented as a beam with reaction forces at the bearings, forces and moments applied at cross-section A (where the crank attaches to the spindle), and a chain tension pulling on the chain ring.

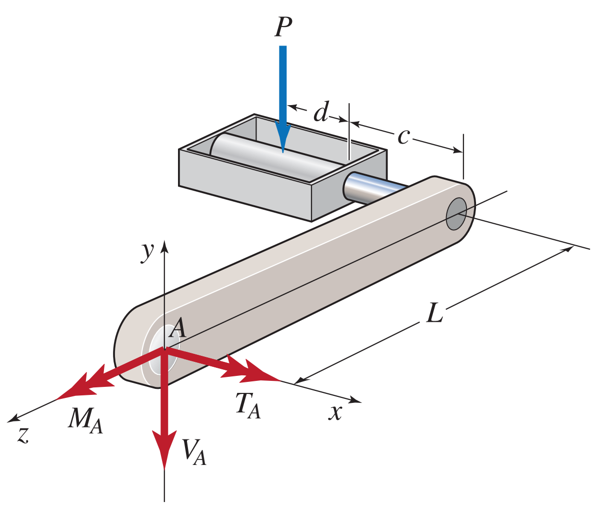


Figure 6: Crank-pedal free-body diagram.

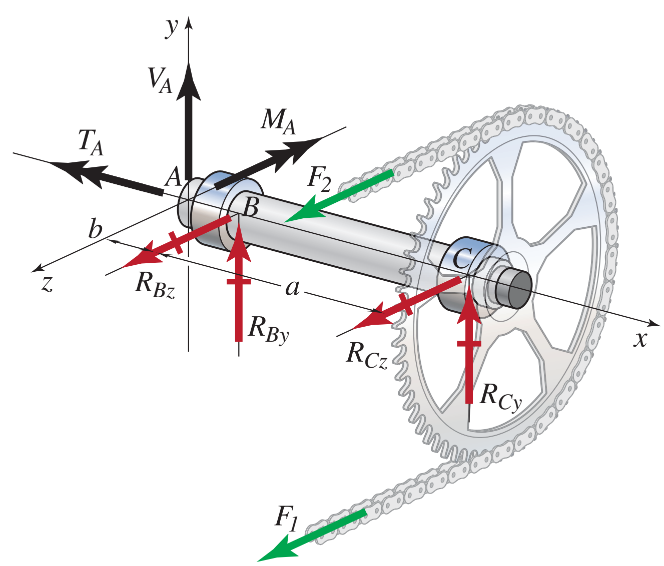


Figure 7: Spindle free-body diagram.

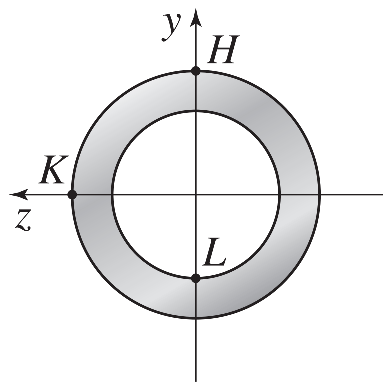
1) As shown in the crank-pedal free-body diagram (Figure 6), pedaling forces bend and twist the spindle. Determine the internal moments and forces at the cross-section A (M_A, T_A and V_A) as a function of the force P. **Note:** We are interested in the internal shear force, moment, and torque of the *spindle*, which has been cut out of this diagram to expose the surface where it meets the crank.

2) Determine the lower chain tension F_1 and the upper chain tension F_2 via the equilibrium equation $(\sum M)^C_{x-axis} = 0$. Assume that the chain ring has radius $R = 75$ mm and lies in the same plane as the right bearing (cross-section C). **Note:** You will need to make one additional assumption regarding the chain tensions.

3) Detemine the reactions at the bearings (R_{By}, R_{Bz}, R_{Cy}, R_{Cz}) as a function of the force P by writing and solving equilibrium force and moment equations.

4) Draw the shear force V_y and bending moment M_z diagrams for the spindle from cross-section A ($x = 0$) to cross-section C ($x = 95$ mm).

5) The spindle cross-section illustrated below is located at $x = 40$ mm. The spindle is hollow with outer diameter $d_o = 27$ mm and inner diameter $d_i = 21$ mm. Assume that $P = 200$ N to determine the **normal stresses** at points H, K and L.



6) The spindle is made of steel with yielding strength $\sigma_Y = 400$ MPa. Determine the maximum force P_{max} that can be applied to the pedal if we want to apply a factor of safety against yielding equal to 2.

7) Use the maximum force P_{max} that you obtained in part (6) to evaluate the maximum shear stress **due to torsion**.