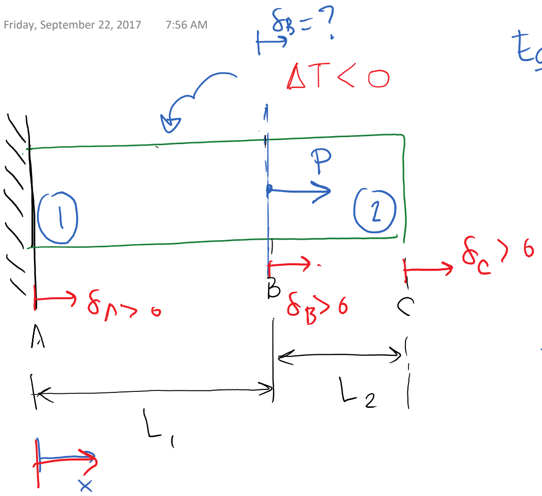
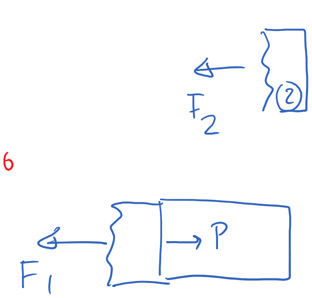
Friday, September 22, 2017 7:56 AM




## Equilibrium




$$\boxed{F_2 = 0} \qquad \boxed{\sigma_2 = 0}$$

$$\boxed{F_1 = P} \qquad \boxed{\sigma_1 = \frac{P}{A}}$$

## Geometry of deformation

$$\boxed{\delta_2 = \delta_C - \delta_B}$$

relative deformation

$$\delta_1 = \delta_B - \cancel{\delta_A}^{\,0}$$

$$\begin{cases} \delta_B = \delta_1 = \dfrac{F_1 L_1}{EA} + \alpha L_1 \Delta T \\[2ex] \delta_C = \delta_2 + \delta_1 = \underbrace{\dfrac{F_2 L}{EA} + \alpha L_2 \Delta T}_{\delta_2} + \underbrace{\dfrac{F_1 L}{EA} + \alpha L_1 \Delta T}_{\delta_1} \end{cases}$$

$F_2 \to 0$

## Force-elongation-temperature relation:

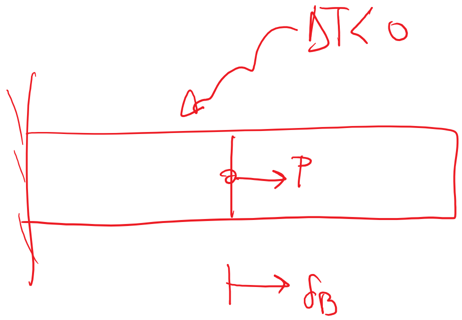
$$\delta = \frac{FL}{EA} + \alpha L \Delta T$$

$$\boxed{\delta_B = + \frac{P L_1}{EA} + \alpha L_1 \Delta T}$$

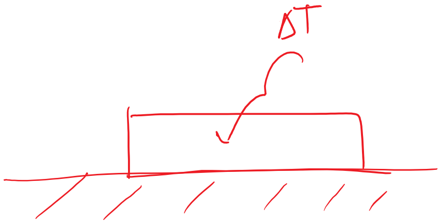
$$\delta_C = \alpha L_2 \Delta T + \frac{P L_1}{EA} + \alpha L_1 \Delta T$$

$$\boxed{\delta_C = \alpha L \Delta T + \frac{P L_1}{EA}}$$

$L \to L_1 + L_2$




$P = 0 \quad \Delta T < 0 \longrightarrow \delta_B < 0$




$\delta_T = \alpha L \Delta T$

$\epsilon_T = \dfrac{\alpha L \Delta T}{L} = \alpha \Delta T$

Force → Stress

Disp → Strain

Stress ↔ Strain

Side note: (not included in lecture)

Deformation: $\delta = \dfrac{FL}{EA} + \alpha L \Delta T$ (divide by L)

$$\underbrace{\epsilon_{total}}_{\text{total strain}} = \underbrace{\frac{\sigma}{E}}_{\epsilon_M \text{ (mechanical strain)}} + \underbrace{\alpha \Delta T}_{\epsilon_T \text{ (thermal strain)}}$$

For all questions below, we are using the convention:

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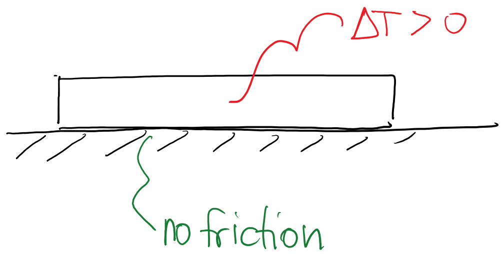
$\delta > 0 \longrightarrow$ elongates $\quad \sigma > 0 \longrightarrow$ tension

$\delta < 0 \longrightarrow$ shorten $\quad \sigma < 0 \longrightarrow$ compression




(1) What happens to the total deformation?

(A) $\delta > 0$

(B) $\delta < 0$

(C) $\delta = 0$

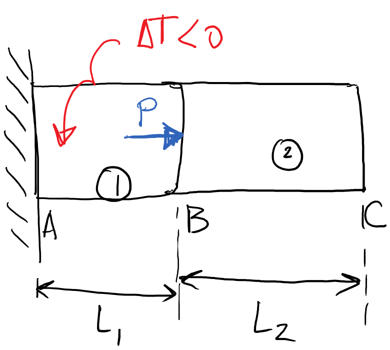
(2) What happens to the stress?

(A) $\sigma > 0$

(B) $\sigma < 0$

(C) $\sigma = 0$

- Fixed at A
- Force applied at B




$E_1 = E_2$

$A_1 = A_2$

(3) Mark the statement that must always be true?

(A) $\delta_1 < 0 \quad \delta_2 > 0$

(B) $\delta_1 + \delta_2 = 0$

(C) $\delta_1 + \delta_2 = \delta_C$

(4) What happens to the stress?

(A) $\sigma_1 \neq 0 \quad \sigma_2 = 0$

(B) $\sigma_1 \neq 0 \quad \sigma_2 \neq 0$

(C) $\sigma_1 = 0 \quad \sigma_2 \neq 0$

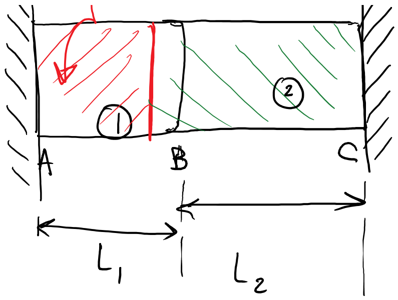
(D) $\sigma_1 = \sigma_2$

- Fixed at A and C


(5) Mark the statement that must always be true?

(A) $\delta_1 < 0 \quad \delta_2 > 0$

(B) $\delta_1 + \delta_2 = 0 = \delta_{total}$




$E_1 = E_2$

$A_1 = A_2$

(B) $\delta_1 + \delta_2 = 0 = \delta_{TOTAL}$

(C) $\delta_1 = \delta_2$

(6) What happens to the stress?

(A) $\sigma_1 \neq 0 \quad \sigma_2 = 0$

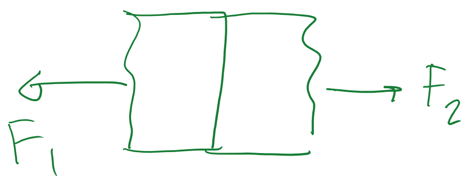
(B) $\sigma_1 \neq 0 \quad \sigma_2 \neq 0$

(C) $\sigma_1 = 0 \quad \sigma_2 \neq 0$

(D) $\sigma_1 = \sigma_2$

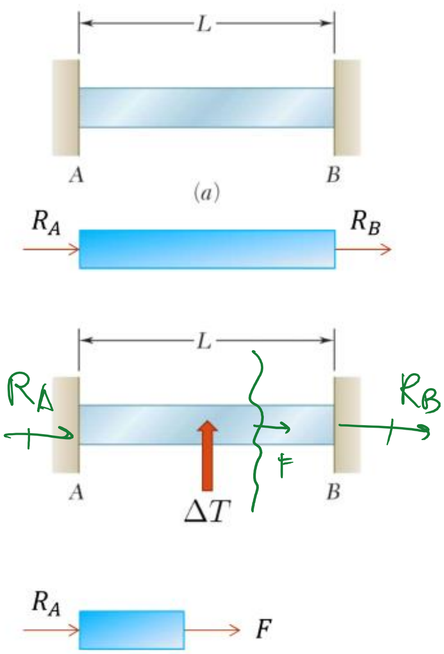
$$\sigma_1 = \frac{F_1}{A}$$

$$\sigma_2 = \frac{F_2}{A}$$



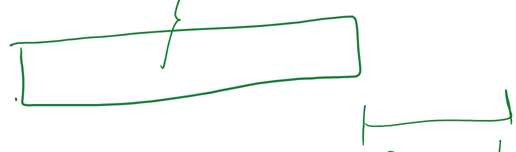

$$\boxed{F_1 = F_2}$$

## Statically indeterminate problems




if $\Delta T > 0 \Rightarrow$ (A) Compressive stresses
(B) Tensile stresses
(C) zero stresses

$\Delta T > 0$



$\delta = \alpha L \Delta T$

$L_f = L + \alpha L \Delta T$

Equilibrium $R_A + F = 0 \Rightarrow \boxed{F = -R_A}$

Geom. def. $\delta_{TOTAL} = 0 \longrightarrow \delta = 0$

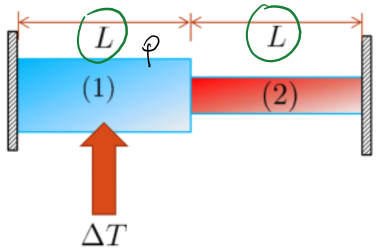
$$\frac{F\cancel{L}}{EA} + \alpha \cancel{L} \Delta T = 0$$

$$\boxed{F = -E\,A\,\alpha\,\Delta T}$$

$\frac{N}{m^2}$ $m^2$ $\frac{1}{{}^\circ\cancel{C}}$ ${}^\circ\cancel{C}$

$\Delta T > 0 \longrightarrow F < 0 \Longrightarrow \sigma < 0$

$$\Delta T < 0 \Longrightarrow F > 0 \Longrightarrow \sigma > 0$$



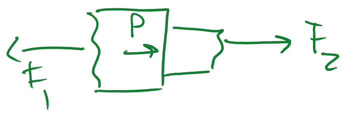

$E_1 = E_2 = E$

$\alpha_1 = \alpha_2 = \alpha$

$A_1, A_2$

Stress

(1) Equil




(2) Geom. def. $\boxed{\delta_1 + \delta_2 = 0}$

(3) F.E.T. relation

$$\delta_1 = \frac{F_1 L}{EA_1} + \alpha L \Delta T$$

$$\delta_2 = \frac{F_2 L}{EA_2} + \cancel{\alpha L \Delta T} \rightarrow 0$$

$$\delta_1 + \delta_2 = 0$$

$$\boxed{\frac{F_1 L}{EA_1} + \alpha L \Delta T + \frac{F_2 L}{EA_2} = 0} \qquad \boxed{F_1 - F_2 = P} \longrightarrow F_1 = P + F_2$$

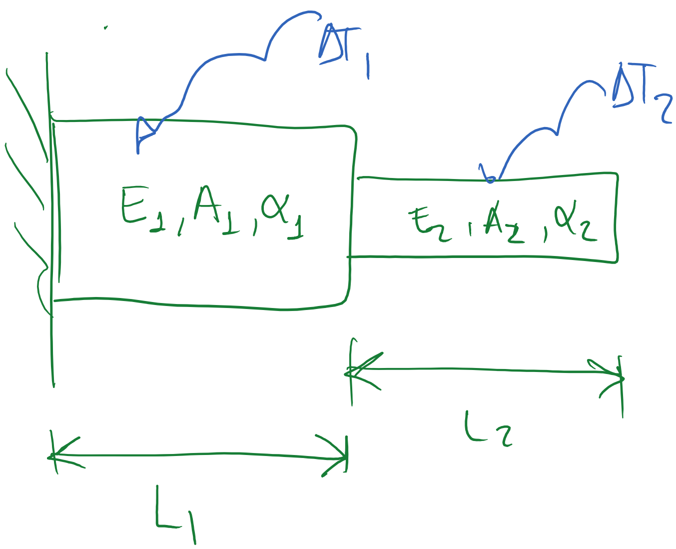
$$(F_2 + P)\frac{L}{EA_1} + \frac{F_2 L}{EA_2} = -\alpha L \Delta T$$

$$F_2\left(\frac{L}{EA_1/A_2} + \frac{L}{EA_2/A_1}\right) = -\alpha L \Delta T - \frac{PL}{EA_1}$$

$$F_2\left(\frac{LA_2 + LA_1}{E A_1 A_2}\right) = -\alpha L \Delta T - \frac{PL}{EA_1}$$

$$\boxed{F_2 = \frac{E A_1 A_2}{(A_1 + A_2)}\left[-\alpha \Delta T - \frac{P}{EA_1}\right]}$$

In general, when all parameters are different ...




$$\delta_1 = \frac{F_1 L_1}{E_1 A_1} + \alpha_1 L_1 \Delta T_1$$

$$\delta_2 = \frac{F_2 L_2}{E_2 A_2} + \alpha_2 L_2 \Delta T_2$$